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spectrum was the same at a given temperature, showing no effect of the reduced possibility of conduction by the vapor. With the first method, an extra protection of the tube from loss of heat permitted the observation of the test spectrum with a potential difference of less than 0.6 volt per centimeter length of the tube.

Two experiments in which there was no potential difference on the tube were next tried. In one, the tube was heated, the current broken, and the exposure made during an interval while the tube cooled with no current. The spectrum was found to be the same as when the furnace was operated in the regular way at the mean temperature of the no-current experiment. In the other test, a tube containing iron was heated by a high-current arc between horizontal electrodes supported beneath the tube. The vapor in the tube used as a crucible in this way emitted a spectrum in which the test lines appeared as in the resistance tube carrying a current.

The conclusion from this series of experiments is that for the temperatures required in the line classification which the writer has carried out for a number of metallic spectra, the potential difference acting on the tube is not effective in modifying the spectrum, the lines sensitive to arc conditions appearing with equal ease whether a voltage is acting on the tube or not. Higher temperatures, accompanied by increase of both ionization and potential drop, should be checked when possible as to the effect of conduction by the vapor. Lines brought out only at these higher temperatures are, however, usually so faint that they are of little importance in the furnace spectrum. For a study involving these lines, the arc would usually be employed.

AN APPLICATION OF THE PORISM OF FOUR TANGENTS OF A TWISTED CUBIC

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Communicated by E. H. Moore, October 25, 1920

The problem of determining the involutions of degree n of a given discriminant has not been solved except when $n = 2,3,4$.

Special forms of discriminant occur in Jacobi's transformation of elliptic integrals and in researches on reducible integrals. Geometrically the problem is to construct a rational curve in n-space of degree n to touch $2(n - 1)$ hyperplanes of a pencil. For $n = 3$ the solution depends on a certain porism;[1] that of four tangents of a twisted cubic. There is in fact the theorem: No proper twisted cubic can be drawn to touch four lines unless they satisfy a condition in which case an infinity of cubics can be drawn. Let *abcd* be four tangents to a twisted cubic meeting their transversals l and l' in $ABCD$, $A'B'C'D'$ and let AA', CC' touch the cubic at points of parameters o, ∞ and BB', DD' at points of parameters θ, φ. It is found that the equations of AA' are $x = y = 0$, of CC' $z = w = 0$, of BB' $x - 2\theta y + \theta^2 z = y - 2\theta z + \theta^2 w = 0$, of DD' $x - 2\varphi y + \varphi^2 z = y - 2\varphi z + \varphi^2 w = 0$. The equations of l are $x - \kappa y = 3z - \lambda w = 0$, of l' $x - \lambda y = 3z - kw = 0$, where k, λ are connected with θ, φ by the equations $\theta + \varphi = \kappa + \lambda/2$, $\theta\varphi = {}^1/_3\, \kappa/\lambda$. Putting $m = \{ABCD\}$, $n = \{A'B'C'D'\}$ the cross-ratios are connected by an equation. The values of m and n are

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[1] Dixon, A. C., on twisted cubics which fulfil certain conditions, *Quar. J. Math.*, **23,** 1889 (343–357).

$$\frac{\varphi}{\theta} \cdot \frac{3\theta - 2\lambda}{3y - 2\lambda}, \quad \frac{\theta}{\varphi} \cdot \frac{3\theta - 2\kappa}{3\varphi - 2\kappa}\,;$$

if $\theta/\varphi = t$ it is found that $mn = t^4$, $m + n = t^4 + 1 - (t - 1)^4$.

Eliminating t; $m^{1/2} - n^{1/2} = 2m^{1/8}n^{1/8}(1 - m^{1/4}n^{1/4})$. This is the poristic condition or if the ranges on the transversals of four lines satisfy this condition an infinity of cubics can be drawn to touch them. The modular equation for the cubic transformation appears here.

To determine the involutions of cubic forms of a given discriminant, let $\alpha\beta\gamma\delta$ be four planes of a pencil of axis l; in $\alpha\beta\gamma$ take lines *abc* arbitrarily (distinct from l) meeting l in distinct points ABC. The cubics tangent to *abc* and the plane δ are of multiplicity ∞^2 being subject to 10 conditions. Let d be the line in δ tangent to a cubic and D its intersection with l and l' the second transversal of *abcd*. If m n have the same meaning as before n is given being the cross-ratio of the pencil $\alpha\beta\gamma\delta$ and m has four values. D has four positions and the lines d form four pencils.

To see that to a value of n correspond only four values of m it suffices to notice that the equation between m and n results from the elimination of t between $mn = t^4$, $m + n = t^4 + 1 - (t - 1)^4$. Considering n as given and eliminating m there results an equation of degree four for t and so four values of m are found.

The final result may be stated in this way. Let $ABCD$ be any four points on the axis of a pencil of planes $\alpha\beta\gamma\delta$ and suppose $\{ABCD\}$ $\{\alpha\beta\gamma\delta\}$ connected by the modular equation; then any four lines, one through each point to lie in the corresponding plane, are tangent to an infinity of twisted cubics.

There are four inequivalent involutions determined on these cubics by the pencil of planes, and n is the invariant of their discriminant. Their construction is as follows: Given n, determine m by the modular equation. From $\theta + \varphi = {}^1/_2(\kappa + \lambda)$, $\theta\varphi = {}^1/_3\kappa\lambda$ and

$$m = \frac{\theta}{\varphi} \cdot \frac{3\theta - 2\lambda}{3\varphi - 2\lambda}$$

$\kappa\lambda$ are determined uniquely. For $mn = t^4$, $m + n = t^4 + 1 - (t - 1)^4$

regarded as equations in t have a unique common solution, φ may be arbitrarily taken and $\theta = t\varphi$. The equations of l are $x - \kappa y = 3z - \lambda w = 0$ and the cubic is $x = \sigma^3$, $y = \sigma^2$, $z = \sigma$, $w = 1$; or finally the involution is $\sigma^3 - \kappa\sigma^2 + \mu(3\sigma - \lambda)$ a single one determined by each value of m.

Putting $X = x - \kappa y$, $Y = x - \lambda y$, $Z = 3z - \lambda w$, $W = 3z - \kappa w$ the planes $X = 0$, $Y = 0$, $Z = 0$, $W = 0$ are the faces of the tetrahedron whose edges are a, c, l, l'. The collineations ($X' = \rho_1 X$, $Y' = \rho_2 Y$, $Z' = \rho_1 Z$, $W' = \rho_2 W$) leave invariant the lines $abcdll'$ evidently but transform any cubic tangent to $abcd$ into an infinity of others.

The coincidence of the equation between m and n with the modular equation leads to this geometrical theorem. If four planes of a pencil of axis l touch a cubic and the tangents meet l in $ABCD$ and if the planes meet the cubic in simple intersections $A'B'C'D'$ then $\{A'B'C'D'\} = \{ABCD\}$.

STUDIES ON THE TRANSFORMATION OF THE INTESTINAL FLORA, WITH SPECIAL REFERENCE TO THE IMPLANTATION OF BACILLUS ACIDOPHILUS, II. FEEDING EXPERIMENTS ON MAN

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In a recent communication in these Proceedings 6, 1920, pp. 423–426 the results of feeding experiments with albino rats were presented. Similar attempts to implant *Bacillus acidophilus* in man have led to the following conclusions.

Thorough implantation may be effected by the daily administration of 300–400 grams of lactose, 300 cubic centimeters of a whey broth culture of *B. acidophilus* (McFarland nephelometer turbidity scale of 5), or by a combination of 150 grams of lactose and 150 cc. of the whey broth culture. Complete transformation of bacterial types may be brought about also by the daily use of 500–1000 cc. of acidophilus milk culture.

The same results are obtained with dextrin as with lactose, but dextrose, sucrose and maltose exert no flora-transforming influence.

The time required for changing the flora varies with the nature and the amounts of the material administered. The most rapid implantation was obtained with the *Bacillus adophilus* milk. In two of the subjects 500 cc. of the milk brought about marked alteration within two to three days. Some of the subjects that were but slightly affected by 500 cc. of the milk culture showed very pronounced transformation when 100 grams of lactose were added daily to this amount.

No change could be detected in the hydrogen ion concentration of the